Physics 0

§1. Introduction and history of Physics

- Ancient Greece (~400 - 300 BC)

  "Philosophy" (love of wisdom)

  ↓

  "physis" (Knowledge of nature)

  ↓ Aristotle

  "Physics", "On the Heavens", "Meteorology"

  → "Metaphysics" (principles of physical science)

Main question: Why are things as we find them, and why they could not be otherwise?

→ Aristotle established logical rules by which causes could be determined from the effects we perceive in nature

"Physics" $\xleftrightarrow{\text{distinct!}}$ "Cosmology"

"Physics"	"Cosmology"
(earthly matters)	(heavenly matters)
change/motion	no change (except for motion in perfect circles)

- Scientific revolution (~ 1605 – 1644 AD)

  main person: Galileo Galilei

  main new ingredient: use of <u>experiment</u> to deduce "laws" of physics

  shift from cause $\longrightarrow$ laws of motion

<u>Basic principles:</u>

1) there basis laws which determine motions of matter
2) these laws are universal and independent of time and place
3) the laws have to be consistent with experiment

4) the laws are formulated using mathematics
(Galileo: "The book of nature is written in the language of mathematics")

5) laws are not absolute or perfect but have an approximate character and are subject to change as our understanding of nature improves

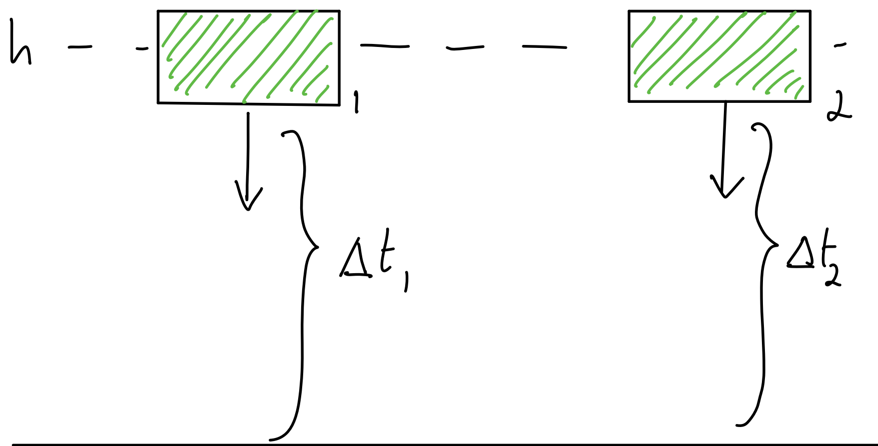
→ contradicts Aristotle's physics

Example 1:

Aristotle: "If a weight falls from a certain height in so much time, a weight which is twice as great will fall from the same height in half the time"

Galileo disproves this as follows:

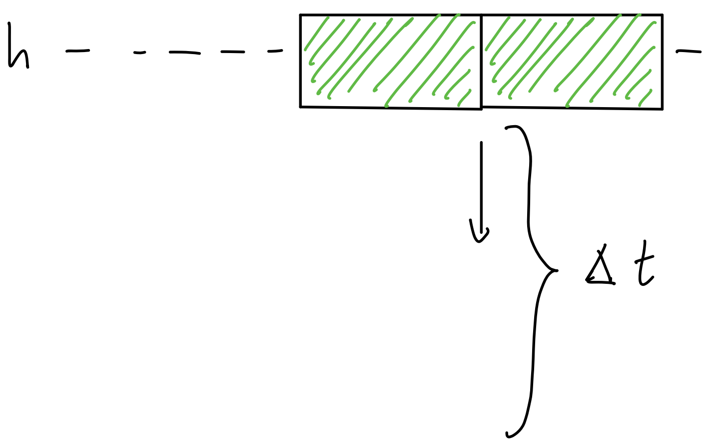
Imagine two bricks with exactly same are falling down from the same height h



$\Longrightarrow \Delta t_1 = \Delta t_2$

(follows from principle 2))

Now imagine we move the brick closer to each other until they touch (we may glue them together)



$\Longrightarrow \Delta t = \Delta t_1 = \Delta t_2$

(follows from principle 2))

Thus it follows that a brick with twice the weight will fall in the <u>same</u> time!

Galileo then proceeds to confirm this with an actual experiment

§1.1 Motion

In physics, in order to formulate laws, one makes simplifications or "abstractions":
We are going to study an object

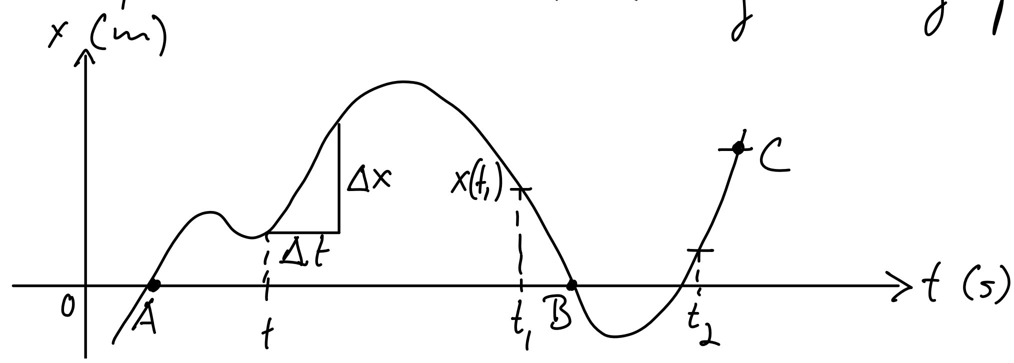
- that is a mathematical point,
- focus first on objects that only move along the x-axis

$\longrightarrow$ pick an origin, $x = 0$

$\longrightarrow$ to describe position, we need units! choose "meter"

$\longrightarrow$ as unit of time choose "second"

Next, describe motion by a graph



Definition 1:

i) The "average velocity" of an object is given by

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

where $t_2 > t_1$ are two times between which we take the average.

ii) The velocity at a given time or "instantanteous velocity", $v(t)$ is defined as

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \text{"differentiation"}$$

iii) The "average acceleration" $\bar{a}$ is defined as

$$\bar{a} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$$

and "instantaneous acceleration" is given by

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

§1.2 Motion at constant acceleration

set $a(t) = a = \text{const.}$

We will see that this is relevant for motion of falling bodies where

$a = -9.8\ \text{ms}^{-2} = -g$

Proplem: guess a function $x(t)$ with $\frac{d^2x}{dt^2} = a$

$\rightarrow$ "integration"

Most general solution:

$$x(t) = \frac{1}{2}at^2 + bt + c \qquad \text{where } b, c = \text{const.}$$

For vertical motion we use symbol y:

$$y(t) = \frac{1}{2}at^2 + bt + c$$

for example $y(t) = -\frac{1}{2}g\,t^2 + bt + c$ for falling bodies

$\rightarrow$ fix initial height and velocity:

$$\left.\begin{array}{l} \cdot\ y(0) = y_0 \\ \cdot\ v(0) = v_0 \end{array}\right\} \Rightarrow y_0 = 0 + 0 + c$$

$\rightarrow$ c is "initial coordinate"

Moreover, $v(t) = \frac{dy}{dt} = -gt + b$

$\Rightarrow \quad v_0 = b$ "initial velocity"

$\rightarrow \quad y(t) = -\frac{1}{2}gt^2 + v_0 t + y_0$

Now consider a general motion with constant acceleration:

$$x(t) - x_0 = \frac{1}{2}at^2 + v_0 t \qquad (*)$$

$$\rightarrow \quad v(t) = at + v_0$$

$$\Leftrightarrow \quad t = \frac{v(t) - v_0}{a}$$

$\rightarrow$ inserting into $(*)$ gives

$$x(t) - x_0 = \frac{1}{2}a\left[\frac{v(t) - v_0}{a}\right]^2 + v_0\left[\frac{v(t) - v_0}{a}\right]$$

$$= \frac{v^2(t) - v_0^2}{2a}$$

$$\Leftrightarrow v^2 - v_0^2 = 2a(x - x_0)$$

<u>Derivation using calculus:</u>

$$\frac{dv}{dt} = a \xrightarrow{\cdot v} v\frac{dv}{dt} = a\frac{dx}{dt} \xrightarrow{\cdot dt} v\,dv = a\,dx$$

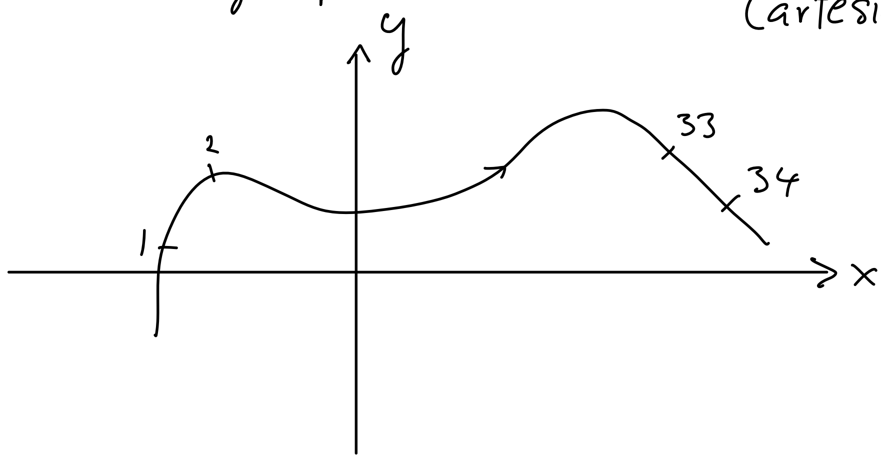
Now integrate: $\int_{v_1}^{v_2} v\,dv = a\int_{x_1}^{x_2} dx$

giving $\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = a(x_2 - x_1)$ $\square$

§2. Motion in higher dimensions

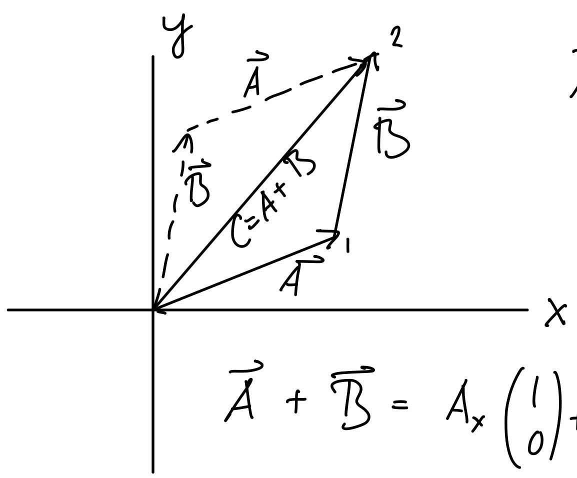
§2.1 Vectors in $d=2$

Consider a particle traveling in the x-y plane: "Cartesian coordinates"



Notice: time axis not shown!
→ label points of graph by time coordinate

We will use "vectors"



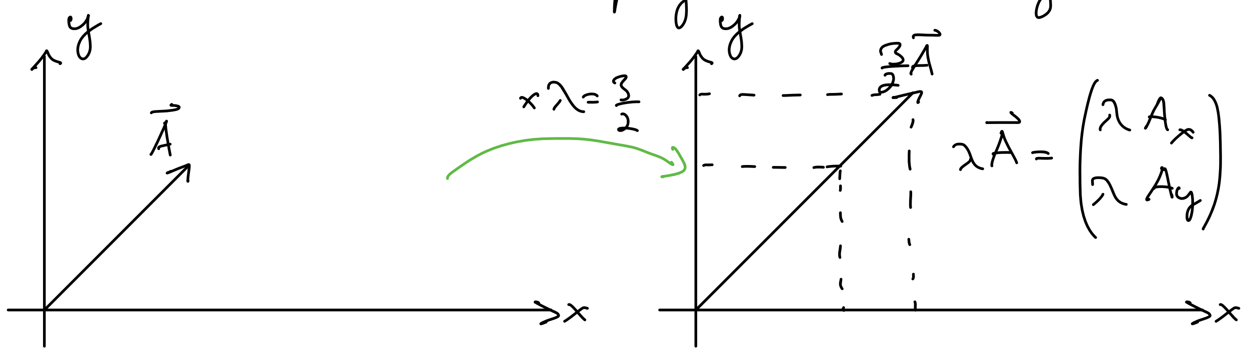
$$\vec{A} = A_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + A_y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

basis vectors

$$\vec{A} + \vec{B} = A_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + A_y \begin{pmatrix} 0 \\ 1 \end{pmatrix} + B_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + B_y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} A_x + B_x \\ A_y + B_y \end{pmatrix}$$

"Rescaling":

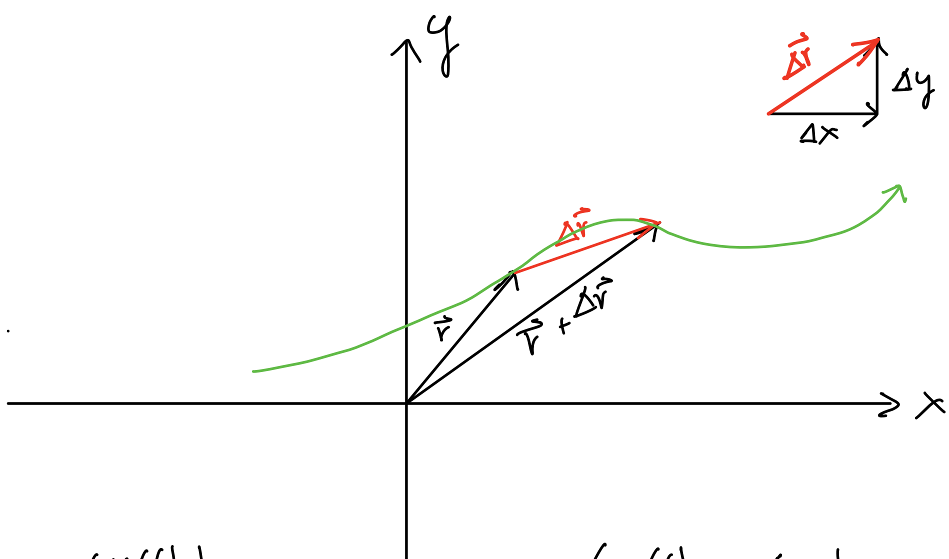
One can also multiply a vector by $\lambda \in \mathbb{R}$:



$$\lambda \vec{A} = \begin{pmatrix} \lambda A_x \\ \lambda A_y \end{pmatrix}$$

§ 2.2 Derivatives of position vector

Consider a particle in the x-y plane that moves from position $\vec{r}$ at time t to $\vec{r} + \Delta\vec{r}$ at time $t + \Delta t$:



$$\vec{r} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \vec{r} + \Delta\vec{r} = \begin{pmatrix} x(t) + \Delta x \\ y(t) + \Delta y \end{pmatrix}, \quad \Delta\vec{r} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$\rightarrow \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} \rightarrow \text{velocity is a vector!}$$

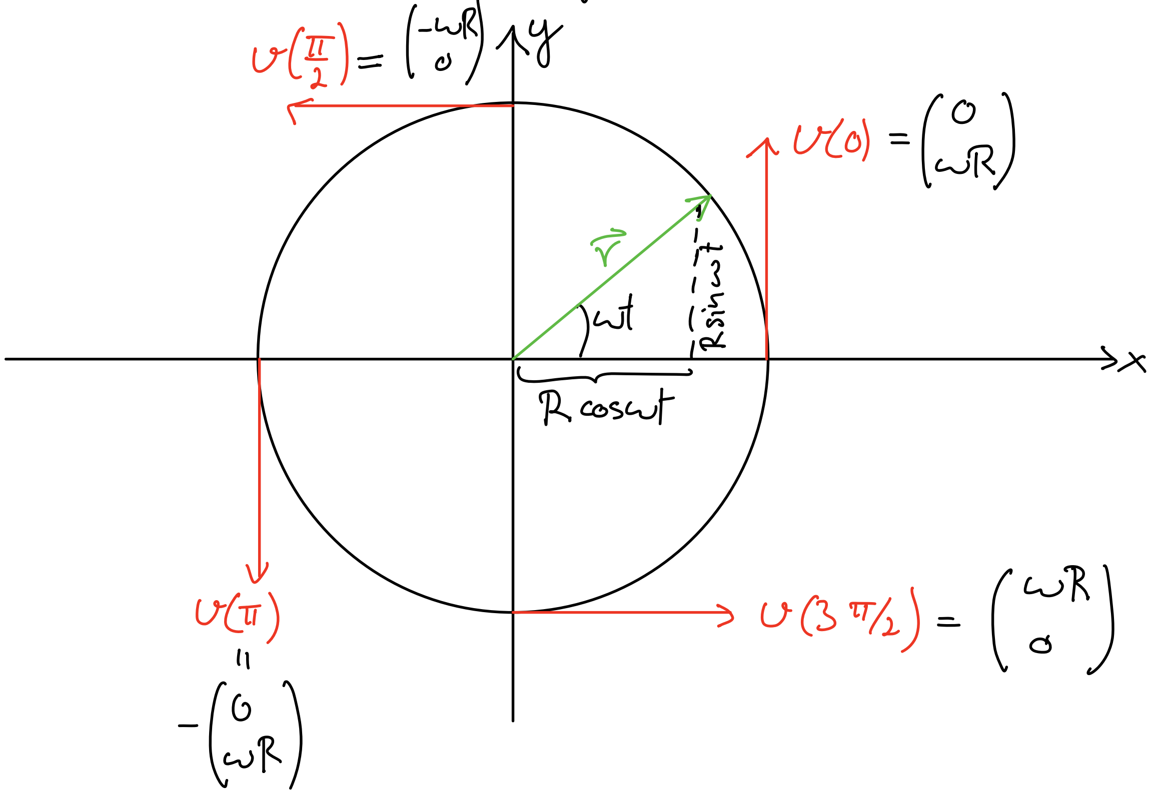
Example 1 (Application to circular motion):

Let us consider motion along a circle with radius R on the x-y plane:

$$\vec{r}(t) = R\left(\cos\omega t\, \vec{e}_1 + \sin\omega t\, \vec{e}_2\right), \quad \vec{e}_1 = \begin{pmatrix}1\\0\end{pmatrix}, \ \vec{e}_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

where R, ω are constants ($\in \mathbb{R}$)

$$\longrightarrow \quad |\vec{r}(t)|^2 = r_x^2 + r_y^2 = R^2(\cos^2\omega t + \sin^2\omega t) = R^2$$



As t increases, ωt increases, and the particle will come back at time T such that

$$\omega T = 2\pi \qquad (\text{using "radians": } 2\pi \cong 360°)$$

$\longrightarrow \quad \omega = \frac{2\pi}{T} = 2\pi f ,$

where

- $f = \frac{1}{T}$ is called "frequency"

  = number of cycles per second
  (measured in Hz "Herz")
- $\omega = 2\pi f$ is called "angular velocity"

Question: How fast is our particle moving?

$\longrightarrow$ compute velocity:

$$v(t) = \frac{d\vec{r}(t)}{dt}$$

$$= R\left(\vec{e}_1 \frac{d\cos\omega t}{dt} + \vec{e}_2 \frac{d\sin\omega t}{dt}\right)$$

$$= R\omega\left(-\vec{e}_1 \sin\omega t + \vec{e}_2 \cos\omega t\right)$$

At $t=0$, $v(0) = R\omega\cos 0\, \vec{e}_2 = R\omega\, \vec{e}_2$

$\longrightarrow$ particle is moving "straight up"
at speed $v = |\vec{v}| = \omega R$

Magnitude of velocity is constant along every point on circle:

$$v^2 = (\omega R)^2(\sin^2\omega t + \cos^2\omega t) = (\omega R)^2 \Rightarrow v = \omega R$$

How about acceleration ?

Let's make a calculation :

$$\vec{a} = \frac{d\vec{v}(t)}{dt} = -\omega^2 R \left(\vec{e}_1 \cos\omega t + \vec{e}_2 \sin\omega t \right)$$
$$= -\omega^2 \vec{r}(t)$$

$$\Rightarrow \quad a = |\vec{a}| = \omega^2 R$$

<u>Interpretation:</u>

When a particle moves in a circle of radius R at constant speed v, it has an acceleration, called "centripetal acceleration", directed toward the center and of magnitude

$$a = \omega^2 R = \frac{(\omega R)^2}{R} = \frac{v^2}{R}$$